Novel Excitonic States in Quantum Hall Systems: Bound States of Spin Waves and a Valence Band Hole

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Abstract

If the Zeeman energy is small, the lowest energy excitations of a two dimensional electron gas at filling factor $\nu = 1$ are spin waves (spin flip excitations). At ν slightly larger (smaller) than unity, reversed spin electrons (spin holes) can form bound states with K spin waves that are known as skyrmions, S_K^- (antiskyrmions, S_K^+). It is suggested in this work that a valence hole can also bind K spin waves to form an excitonic complex X_K^+, analogous to the S_K^+. One spin hole of the S_K^+ is simply replaced by the valence hole. At $\nu \leq 1$, a small number of S_K^+'s are present before introduction of the valence hole. The S_K^+–X_K^+ repulsion leads to correlations and photoluminescence similar to those of a dilute electron–charged-exciton (e–X^-) system at $\nu \leq \frac{1}{3}$. At $\nu \geq 1$, the S_K^-–X_K^+ attraction can potentially lead to different behavior.
Keywords: quantum Hall effect, skyrmion, photoluminescence

1 Introduction

It has become clear [1, 2] that neutral (X) and charged (X^-) excitons both play an important role in the photoluminescence (PL) spectrum of realistic quantum Hall systems at high magnetic field and low electron density (i.e. for filling factor $\nu \leq \frac{1}{3}$). This is true despite the "hidden symmetry" of the ideal theoretical model ("ITM" implies zero well width, w, and very high magnetic field, B; impurity scattering will be ignored in all our calculations) which suggests that PL occurs only from neutral exciton recombination [3]. At values of ν close to unity a considerable body of experimental data exists [4, 5], but no simple picture of the PL process has emerged. In this note we suggest that positively charged excitonic complexes (X_K^+) consisting of K spin waves (SW), each with angular momentum $l_{\rm SW} = 1$, bound to a valence hole (v) must occur for $\nu \approx 1$, and that in real experimental systems at low temperature these X_K^+ complexes

could dominate the PL spectrum. A SW consists of a reversed-spin-electron–spin-hole pair ($e_{\rm R}h$) in the lowest Landau level (LL) of the conduction band.

Throughout this paper we contrast the predictions of the ITM with those of realistic systems. The latter requires the admixture of a number of LL's by the Coulomb interaction and taking into account the finite well width w. Finite separation d between the electron layer and the valence hole layer can also be included. These effects destroy the "hidden symmetry" which occurs when the magnitude $|V_{ij}|$ of the Coulomb interaction is the same for any pair (i,j) selected from $(e_{\rm R}, h, v)$. The paper is organized in three main sections. Section 2 contains a summary of the results predicted [1] for PL in dilute systems ($\nu \leq \frac{1}{3}$). Section 3 section contains a discussion of the elementary spin excitations [6] of a system of N electrons with ν close to unity in the absence of any valence band holes. In Section 4 a valence hole is introduced into the $\nu \approx 1$ system. The formation of X_K^+ ($v + K \times {\rm SW}$) complexes is discussed using their analogy to skyrmions or antiskyrmions. The implication for PL of the existence of a quantum liquid consisting of electrons, skyrmions (antiskyrmions) and an X_K^+ for $\nu \geq 1$ ($\nu \leq 1$) are discussed. Some preliminary numerical results for simple realistic systems are presented.

2 Energy Spectrum and PL for $\nu \leq \frac{1}{3}$

It has become rather standard to diagonalize numerically the Coulomb interaction for a finite system of N electrons confined to a spherical surface which contains at its center a magnetic monopole of strength $2Q$ flux quanta [7]. In the ITM only states of the lowest LL are included. For realistic experimental systems (having a finite quantum well width w in a finite magnetic field B) both higher LL's and the modification for the Coulomb matrix elements associated with the envelope functions of the quantum well must be included.

In Fig. 1 we present the energy spectrum for simple system consisting of two electrons and one valence band hole at $2Q = 20$ evaluated in the ITM and excluding the Zeeman energy [1]. The solid dots are triplet electron states (the total spin of the pair of electrons $S = 1$); the open circles are singlets ($S = 0$). The state labeled $e+X$ at angular momentum $L = 10$ is a "multiplicative state" consisting of an unbound electron and a neutral exciton (X). Notice that only one bound state (labeled $X_{\rm td}^-$) occurs. It is at $L = 9$ and is called the "dark triplet" because it is forbidden to decay radiatively.

In Fig. 2 similar results are presented for a realistic system consisting of a symmetric GaAs quantum well of width $w = 11.5$ nm at the finite values of the magnetic field $B = 68$ and 13 T. The appropriate electron Zeeman splitting has been included, and only the lowest state of each triplet is shown. To achieve even qualitative agreement with experimental data, it has also been necessary to include a number of higher LL's, particularly at the lower magnetic fields. Five LL's were needed to obtain convergence in our calculations. In Fig. 2a, at the high magnetic field of 68 T, the $X_{\rm td}^-$ at $L = 9$ is still the ground state, but singlet and another triplet bound states occur at other values of L (the singlet $X_{\rm s}^-$ at

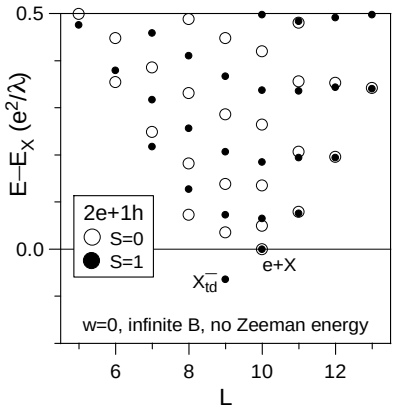

Figure 1: The energy spectrum (energy E vs. angular momentum L) of the $2e$–$1v$ system on a Haldane sphere with the Landau level degeneracy of $2Q+1=21$. E_X is the exciton energy, and λ is the magnetic length. Different symbols distinguish between singlet ($S=0$) and triplet ($S=1$) states.

$L=8$ and the bright triplet $X^-_{\rm td}$ at $L=10$ have roughly half the binding energy of the $X^-_{\rm td}$). At $B=13$ T, as shown in Fig. 2b, $X^-_{\rm s}$ is the ground state, and the $X^-_{\rm td}$ at $L=9$ and X^-_{tb} at $L=10$ are excited states. The spectrum is quite sensitive to the experimental parameters. The well width w enters the Coulomb interaction [1] through $V(r)=e^2/\sqrt{r^2+d^2}$, where d is proportional to w. The cyclotron frequencies $\omega_{ce}(B)$ and $\omega_{cv}(B)$ for the electrons and valence band hole, and the Zeeman energy $E_{\rm Z}(B)$, are taken from experiment, after Refs. [8] and [9], respectively. For the values of the parameters used in our calculations, the singlet and triplet ground states cross at a value of B of the order of 30 T. This is in agreement with the calculations of Whittaker and Shields [10] who used a different numerical approach. Because exact diagonalization gives the eigenfunctions as well as the eigenvalues, it is straightforward to evaluate matrix elements of the luminescence operator $\hat{L}=\int {\rm d}^2r\,\hat{\Psi}_e(r)\hat{\Psi}_v(r)$ between an initial state Φ_i of N electrons and one valence hole, and final states Φ_f containing $N-1$ electrons. $\hat{\Psi}_e$ and $\hat{\Psi}_v$ are the annihilation operators for an electron and valence hole respectively. The oscillator strength for the transition [11] from $|\Phi_i\rangle$ to $|\Phi_f\rangle$ is proportional to $|\langle\Phi_f|\hat{L}|\Phi_i\rangle|^2$. For an isolated X^- (where $N=2$) angular momentum conservation forbids the lowest triplet ($X^-_{\rm td}$) from decaying radiatively; the subscript "d" stands for "dark". The $X^-_{\rm s}$ and $X^-_{\rm tb}$ have finite oscillator strengths which are of the same order of magnitude.

When additional electrons are present ($N>2$) radiative decay of the $X^-_{\rm td}$ is not strictly forbidden, since in the recombination process an unbound electron can scatter, changing the momentum of the final state. However, it was found that for $\nu\le\frac{1}{3}$ such decays are weak because Laughlin correlations of the X^-

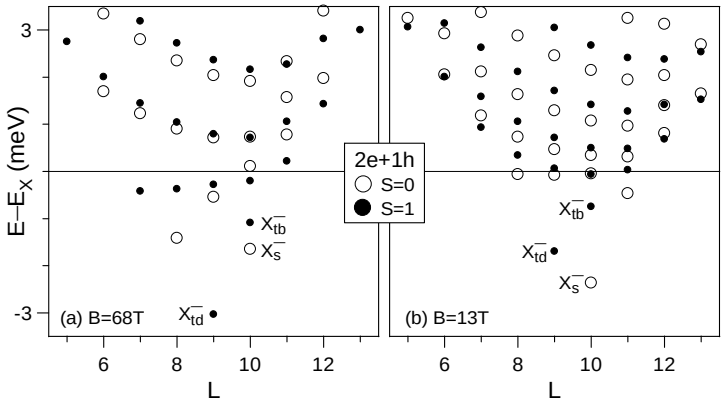

Figure 2: Same as Fig. 1, but for a realistic GaAs quantum well of width $w = 11.5$ nm at the finite values of B as shown. The Zeeman energy has been included, and five LL's for both electrons and hole have been used in the diagonalization.

with unbound electrons inhibit close collisions. The amplitude for radiative decay of the $X_{\rm td}^-$ is estimated [1] to be smaller by one or more orders of magnitude than those of the $X_{\rm s}^-$ and $X_{\rm tb}^-$. It was suggested in [1] that the $X_{\rm td}^-$ would be difficult to see in PL, and that the non-crossing peaks observed by Hayne *et al.* [12] were the $X_{\rm s}^-$ and $X_{\rm tb}^-$. The presence of impurities relaxes the $\Delta L = 0$ selection rule, and the $X_{\rm td}^-$ peak is clearly observed at very low temperature where the excited $X_{\rm tb}^-$ and $X_{\rm s}^-$ states are sparsely populated [2]. The agreement of experiment [2] and the numerical predictions [1] reinforce the hope of using PL to understand correlations in fractional quantum Hall systems.

3 Spin Excitations Near $\nu = 1$

For filling factor ν equal to unity, the lowest energy excitations are spin flip excitations which create a reversed spin electron, $e_{\rm R}$, in the same $n = 0$ LL leaving behind a spin hole, h, in the otherwise filled $\nu = 1$ state. Even when the Zeeman energy $E_{\rm Z}$ is zero, the Coulomb exchange energy will spontaneously break the spin $(\uparrow, \downarrow)$ symmetry giving a spin polarized ground state. In Fig. 3a we show the low lying spin excitations of the $\nu = 1$ state (with $E_{\rm Z}$ taken to be zero) for a system of $N = 12$ electrons [6]. The solid square at $L = 0$ is the spin polarized $\nu = 1$ ground state with spin $S = 6$. The symbol $K = \frac{1}{2}N - S$ is the number of spin flips away from the fully spin polarized state. The band of open squares connected by a dashed line gives the spin wave dispersion $\varepsilon_{\rm SW}(L)$. The angular momentum L is related to wave vector k by $L = kR$, where R is the radius of the spherical surface to which the N electrons are confined. The

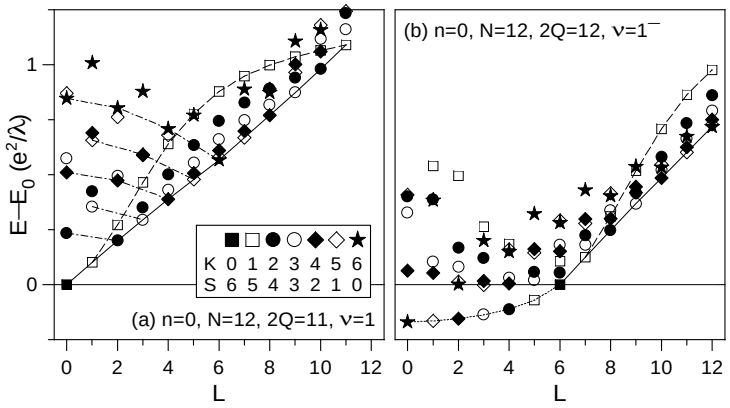

Figure 3: The energy spectra of 12 electrons in the lowest LL calculated on Haldane sphere with $2Q = 11$ (a) and 12 (b).

SW consists of a single $e_{\rm R}h$ pair; its dispersion can be evaluated analytically [13]. The solid circles, open circles, etc. represent states containing 2, 3, ... spin flips (i.e. 2, 3, ... $e_{\rm R}h$ pairs). Dot-dashed lines connect low lying states with equal numbers of spin flips. It is interesting to note the almost straight line connecting the lowest energy states at $0 \leq L \leq 6$. This can be interpreted as band of K SW's each with $l_{\rm SW} = 1$ with $L = K$. The near linearity suggests that these K SW's are very nearly non-interacting. In Fig. 3b we show the numerical results for the situation in which $2Q = 12$, so that one vacancy must be present in the $\nu = 1$ state. The notation is the same as in Fig. 3a. Here the $S = 0$ state appears at $L = 6$. This is simply the single spin hole of $l = Q = 6$ (Q is the angular momentum of the lowest LL or angular momentum shell). What is most interesting in the figure is the band of low lying states containing $K = 0$, 1, 2, ... SW's bound to the spin hole. The energy decreases with increasing K, but the decrease is slower than linear. In Fig. 3 we have neglected the Zeeman energy (taken the g-value equal to zero). For a finite g-value the Zeeman energy is simply $KE_{\rm Z}$, where $E_{\rm Z}$ is the Zeeman energy of a single spin flip. The Coulomb energy of the lowest state containing K SW's is $E_{\rm C}(K) \approx E_{\rm C}(\frac{1}{2}N) + \beta S^2$, where $E(\frac{1}{2}N)$ is the energy of the lowest $L = S = 0$ state in Fig. 3a, and S, the total spin, is equal to $\frac{1}{2}N - K$. Adding the Zeeman energy $KE_{\rm Z}$ leads to a total energy $E(K) = E_{\rm C}(\frac{1}{2}N) + \beta(\frac{1}{2}N - K)^2 + E_{\rm Z}K$. This energy has a minimum at $K = K_0 = \frac{1}{2}(N - E_{\rm Z}/\beta)$ implying that the lowest state contains approximately K_0 spin flips. For $E_{\rm Z} = 0$, $K_0 = \frac{1}{2}N$, and the ground state is completely depolarized (i.e. $S = 0$). As $E_{\rm Z}$ is increased, the number of spin flips, K, in the lowest energy state decreases until at $E_{\rm Z} > \beta N$ only the spin hole in the $\nu = 1$ state remains. The state with the integral value of K (closest to K_0) which gives the lowest energy is a measure of the size of the antiskyrmion, the state consisting of K SW'ss bound to a spin hole in the

$\nu = 1$ state [14]. By electron–hole symmetry the state containing one reversed spin electron, $e_{\rm R}$, in addition to the filled $\nu = 1$ level will form an analogous skyrmion state consisting of K SW'ss bound to the original $e_{\rm R}$.

The most stable skyrmion or antiskyrmion size depends weakly on the quantum well width for the $\nu \approx 1$ state, but for $\nu \approx 3, 5, \ldots$ the well width w must be of the order of a few times the magnetic length in order to obtain stable bound states of SW's and spin holes or reversed spin electrons [6, 15]. As reported by Melik-Alaverdian *et al.* [16], the inclusion of the admixture of higher LL's caused by the Coulomb interaction weakly affects the skyrmion energy spectrum, particularly when the finite wel width w is also taken into account.

The skyrmion and antiskyrmion states $S_K^{\pm}$ are quite analogous to the excitonic $X_K^{\pm}$ states of valence band holes interacting with conduction band electrons. In the ITM, a valence hole has exactly the same interactions as a spin hole in the $\nu = 1$ state of the conduction band. In fact these two types of holes can probably be distinguished by an isospin as is done for electrons on different layers of a bilayer system [17]. The spectrum and possible condensed states of a multicomponent Fermion liquid containing electrons, X_1^-, X_2^-, ..., etc., has been considered by Wójs *et al.* [18]. Exactly the same ideas are applicable to a liquid of electrons and skyrmions or antiskyrmions of different sizes. The only difference is that the skyrmion $S^- = he_{\rm R}^2$ is stable while the $X^- = ve^2$ has a finite lifetime for radiative recombination of an electron–valence-hole pair.

When there are N_h spin holes in the $\nu = 1$ level (or N_e reversed spin electrons in addition to the filled $\nu = 1$ level) and when N_h (or N_e) is much smaller than $N \approx 2Q + 1$, the degeneracy of the filled lowest LL, then the most stable configuration will consist of N_h antiskyrmions (or N_e skyrmions) of the most stable size. These antiskyrmions (or skyrmions) repel one another. They are positively (or negatively) charged Fermions with standard LL structure, so it is not surprising that they would form either a Wigner lattice or a Laughlin condensed state with ν for the antiskyrmion (or skyrmion) equal to an odd denominator fraction as discussed in Refs. [6, 19, 20].

4 Photoluminescence Near $\nu = 1$

In the ITM, a valence hole acts exactly like a spin hole in the $\nu = 1$ level of the conduction band. Therefore we would expect an excitonic complex consisting of K SW's bound to the valence hole to be the lowest energy state, in the same way that the antiskyrmion consisting of K SW's bound to a spin hole in the $\nu = 1$ level gives the lowest energy state when $E_{\rm Z}$ is less than βN. For a small number of valence holes, the $X_K^+ = v(e_{\rm R}h)^K$ excitonic complexes formed by each valence hole will repel one another. If a small number of antiskyrmions are already present (for $\nu < 1$), the positively charged antiskyrmion–charged-exciton repulsion will lead to Laughlin correlations or Wigner crystallization of the multicomponent Fermion liquid. Just as for the X^- excitons in the dilute regime, the PL at low temperature will be dominated by the $X_K^+ \rightarrow S_{K'}^+ + \gamma$ process, with $K' = K$ or $K-1$ depending on spin of the annihilated valence hole

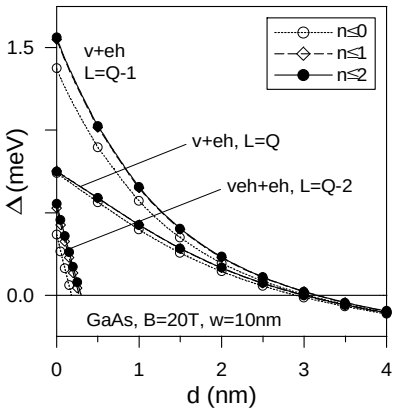

Figure 4: Binding energies of $X_1^+ = v(e_{\rm R}h)$ at $L = Q - 1$ and Q, and of $X_2^+ = v(e_{\rm R}h)^2$ at $L = Q - 2$ as a function of d, the e–v layer separation. The calculations are for a GaAs quantum well of width 10 nm at a magnetic field $B = 20$ T. Different curves include one, two, and three LL's for the valence hole.

(i.e. on the circular polarization of the emitted photon γ). This corresponds to the most stable X_K^+ undergoing radiative ev or $e_{\rm R}v$ recombination and leaving behind an antiskyrmion consisting of K or $K - 1$ SW's bound to a spin hole of the $\nu = 1$ state. Because the valence hole and the spin hole in the $\nu = 1$ conduction level are distinguishable (or have different isospin) even in the ITM this PL is not forbidden. It will be very interesting to see how realistic sample effects (finite well width, LL admixture, finite separation between the electron and valence hole layers) alter the conclusions of the ITM.

For $\nu \geq 1$, negatively charged skyrmions are present before the introduction of the valence holes. The skyrmions are attracted by the X_K^+ charge exciton, but how this interaction affects the PL can only be guessed. We are currently investigating real sample effects in systems containing a small number of skyrmions (or antiskyrmions) and valence band excitonic complexes. As one preliminary example we show in Fig. 4 the binding energy of the $X_1^+ = v(e_{\rm R}h)$ and $X_2^+ = v(e_{\rm R}h)^2$ complexes for different values of the total angular momentum L as a function of the separation between the electron and valence hole layers. The calculation was done for parameters corresponding to a GaAs quantum well of width $w = 10$ nm, at a magnetic field of 20 T. Different symbols (open circles, open diamonds, and solid circles) are for calculations in which one, two, or three LL's for the valence hole have been included (inter-LL excitations of conduction electrons are less important due to their smaller effective mass. Clearly, binding energies decrease with increasing layer separation as expected.

We believe that numerical diagonalization for realistic models including LL admixture and finite well width should explain the behavior of PL for electron

filling factors close to unity. The qualitative behavior expected has been discussed in this note. Realistic "numerical experiments" are being carried out to check whether the expected behavior is correct. These results will be reported elsewhere.

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